Signals and Systems (NTUT-EE 320097)

Midterm Practice

NOT an assignment and NO NEED to hand in your answers

1. A discrete-time signal $x[n]$ is shown in Fig. 1. Please sketch the following signals:

   (a) $x_1[n] = x[n-4]$

   (b) $x_2[n] = x[n-1] + x[n] + x[n+1]$

   (c) $x_3[n] = x[3n+1]$

   (d) $x_4[n] = u[n]x[3-n]$

   (e) $x_5[n] = \frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$

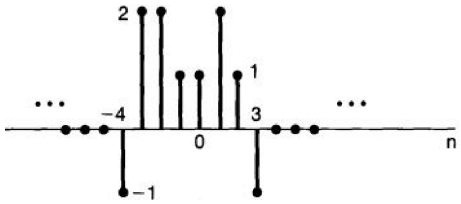

Figure 1: $x[n]$

2. Two discrete-time signals, $w[n]$ and $v[n]$, are given in Figs. 2 and 3, respectively. Please find the odd signal corresponding to each of the following signals：

   (a) $w[n]$

   (b) $w[3 - n/2]$

   (c) $v[n]$

   (d) $w[n] + v[n]$

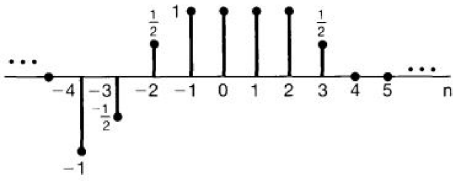

Figure 2: $w[n]$

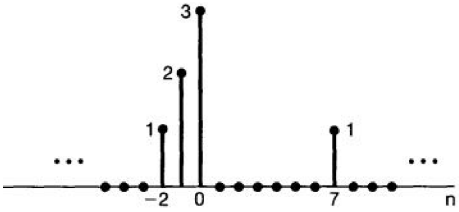

Figure 3: $v[n]$

3. Consider a continuous-time signal $x(t)$ and let $x_o(t)$ and $x_e(t)$ be respectively the corresponding odd and even signals. Please prove the following properties.

   (a) $\int_{-\infty}^{\infty} x_o(t)\, dt = 0$

   (b) The product of $x_o(t)$ and $x_e(t)$ would be an odd signal.

   (c) $\int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} x_o^2(t)\, dt + \int_{-\infty}^{\infty} x_e^2(t)\, dt$.

4. For each of the following input-output relationships, determine whether the corresponding system is linear, time invariant or both.

   (a) $y(t) = t^2 x(t-1)$

   (b) $y[n] = x[n+2] - x[n-3]$

   (c) $y[n] = O_d\{x[n]\}$.

5. Please determine whether or not each of the following continuous- or discrete-time signals is periodic. If the signal is periodic, please find its fundamental period.

   (a) $x(t) = 3\cos(4t + \pi/3)$

   (b) $x(t) = O_d\{\sin(4\pi t)u(t)\}$ (i.e., the odd-signal component of the signal $\sin(4\pi t)u(t)$)

   (c) $x[n] = \cos(\frac{n}{8} - \pi)$

6. A system can be described by the relation between the input $x(t)$ ($x[n]$) and the associated output $y(t)$ ($y[n]$), and it may or may not be memoryless, time invariant, linear, causal, and stable. Please determine which of these properties hold and which do not hold for each of the following systems.

   (a) $y[n] = E_v\{x[n-1]\}$

   (b) $y[n] = x[n-2] - 2x[n-3]$

   (c) $y(t) = \cos(3t+2)x(t)$

   (d) $y(t) = \begin{cases} 0, & \text{if } t < 0 \\ x(t) + x(t-2), & \text{otherwise} \end{cases}$

7. Let '$*$' denote the convolution operation. Please find $y(t)$ or $y[n]$ where $y(t) = x(t) * h(t)$ and $y[n] = x[n] * h[n]$.

   (a) $x(t) = \begin{cases} t, & \text{if } 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$, and $h(t) = \delta(t-1) + \delta(t+1)$.

   (b) $x[n] = \sum_{k=0}^{3} \delta(n-2k)$, and $h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$.

   (c) $x[n] = \begin{cases} 1, & \text{if } 3 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$, and $h[n] = \begin{cases} 1, & \text{if } 4 \le n \le 12 \\ 0, & \text{otherwise} \end{cases}$.